

## SECTION 4.4: GRAPHING FUNCTIONS

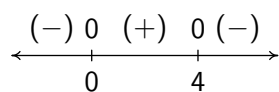
To graph an amazingly detailed graph of a function  $y = f(x)$ ...

1. Find the domain of  $f$ .
2. Simplify the expression  $f(x)$ , if possible.
3. Use limits to analyze  $f$  near any values excluded from the domain of  $f$ .
4. Find the  $x$ - and  $y$ -intercepts of the graph of  $y = f(x)$ .
5. Check for symmetry (even/odd):
  - Recall:  $f$  is even if  $f(-x) = f(x)$  for all  $x$  in the domain of  $f$ .  
The graph of even functions are symmetric about the  $y$ -axis.
  - Recall:  $f$  is odd if  $f(-x) = -f(x)$  for all  $x$  in the domain of  $f$ .  
The graph of odd functions are symmetric about the origin.
6. Use limits as  $x \rightarrow \pm\infty$  to determine the end behavior of  $f$ .
7. Use  $f'(x)$  to find the intervals over which  $f$  is increasing / decreasing; locate any relative (local) extrema.
8. Use  $f''(x)$  to find the intervals over which  $f$  is concave up / down; locate any inflection points.

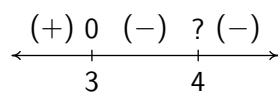
**EXAMPLE 1: (VIDEO)** Consider the function  $f$  whose information is given below:

- $f$  is continuous on  $(-\infty, \infty)$ .

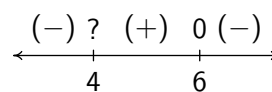
- $\lim_{x \rightarrow -\infty} f(x) = -\infty$  and  $\lim_{x \rightarrow \infty} f(x) = -\infty$



A sign diagram for  $f(x)$



A sign diagram for  $f'(x)$



A sign diagram for  $f''(x)$

1. List the open interval(s) over which  $f$  is:

- increasing:
- decreasing:
- concave up:
- concave down:

2. Determine the  $x$ -value(s) (if any!) where  $f$  has the following features:

- local maximum:
- local minimum:
- absolute maximum:
- absolute minimum:
- inflection point:

3. Sketch a possible graph of  $y = f(x)$ :

**EXAMPLE 2: (VIDEO)** For  $f(x) = 6x\sqrt[3]{20-x}$ ,  $f'(x) = \frac{8(15-x)}{(20-x)^{2/3}}$ , and  $f''(x) = \frac{8(x-30)}{3(20-x)^{5/3}}$ .

1. Find the  $x$ - and  $y$ -intercepts of the graph of  $y = f(x)$ .
2. Find  $\lim_{x \rightarrow -\infty} f(x)$  and  $\lim_{x \rightarrow \infty} f(x)$  to determine the end behavior of  $f$ .
3. Use  $f'(x)$  to determine the intervals over which  $f$  is increasing or decreasing.
4. Use  $f''(x)$  to determine the intervals over which  $f$  is concave up or down.
5. Graph  $y = f(x)$ . Label the intercepts, the relative (local) extrema, inflection points, and vertical tangents.

**EXAMPLE 3: (VIDEO)** Let  $f(x) = \frac{4x}{\sqrt{x^2-1}}$ .

1. Find the domain of  $f$ .
2. Why are there no  $x$ - or  $y$ -intercepts on the graph of  $y = f(x)$ ?
3. Use limits to determine the behavior of  $f$  near  $x = -1$  and  $x = 1$ .
4. Find  $\lim_{x \rightarrow -\infty} f(x)$  and  $\lim_{x \rightarrow \infty} f(x)$  to determine the end behavior of  $f$ .
5. Find and simplify  $f'(x)$  and use  $f'(x)$  to determine the intervals over which  $f$  is increasing or decreasing.
6. Find and simplify  $f''(x)$  and use  $f''(x)$  to determine the intervals over which  $f$  is concave up or down.
7. Graph  $y = f(x)$ . Label the intercepts, relative (local) extrema, inflection points, and vertical tangents.

**HOMEWORK:** Section 4.4: 9 - 57 every other odd.