SECTION 4.4: GRAPHING FUNCTIONS

To graph an amazingly detailed graph of a function $y = f(x)$...

1. Find the domain of f.
2. Simplify the expression $f(x)$, if possible.
3. Use limits to analyze f near any values excluded from the domain of f.
4. Find the $x-$ and $y-$intercepts of the graph of $y = f(x)$.
5. Check for symmetry (even/odd):
   - Recall: f is even if $f(-x) = f(x)$ for all x in the domain of f.
     The graph of even functions are symmetric about the y-axis.
   - Recall: f is odd if $f(-x) = -f(x)$ for all x in the domain of f.
     The graph of odd functions are symmetric about the origin.
6. Use limits as $x \to \pm\infty$ to determine the end behavior of f.
7. Use $f'(x)$ to find the intervals over which f is increasing / decreasing; locate any relative (local) extrema.
8. Use $f''(x)$ to find the intervals over which f is concave up / down; locate any inflection points.

EXAMPLE 1: (VIDEO) Consider the function f whose information is given below:

- f is continuous on $(-\infty, \infty)$.
- $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = -\infty$

$(-)\ 0\ (+)\ 0\ (-)$ at $x = 0$ and $x = 4$

A sign diagram for $f(x)$

$(+)\ 0\ (-)\ ?\ (-)$ at $x = 3$ and $x = 4$

A sign diagram for $f'(x)$

$(-)\ ?\ (+)\ 0\ (-)$ at $x = 4$ and $x = 6$

A sign diagram for $f''(x)$

1. List the open interval(s) over which f is:
   - increasing:
   - decreasing:
   - concave up:
   - concave down:

2. Determine the x-value(s) (if any!) where f as the following features:
   - local maximum:
   - local minimum:
   - absolute maximum:
   - absolute minimum:
   - inflection point:

3. Sketch a possible graph of $y = f(x)$:

EXAMPLE 2: (VIDEO) For $f(x) = 6x\sqrt[3]{20-x}$, $f'(x) = \dfrac{8(15-x)}{(20-x)^{2/3}}$, and $f''(x) = \dfrac{8(x-30)}{3(20-x)^{5/3}}$.

1. Find the x- and y-intercepts of the graph of $y = f(x)$.
2. Find $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to\infty} f(x)$ to determine the end behavior of f.
3. Use $f'(x)$ to determine the intervals over which f is increasing or decreasing.
4. Use $f''(x)$ to determine the intervals over which f is concave up or down.
5. Graph $y = f(x)$. Label the intercepts, the relative (local) extrema, inflection points, and vertical tangents.

EXAMPLE 3: (VIDEO) Let $f(x) = \dfrac{4x}{\sqrt{x^2-1}}$.

1. Find the domain of f.
2. Why are there no x- or y-intercepts on the graph of $y = f(x)$?
3. Use limits to determine the behavior of f near $x = -1$ and $x = 1$.
4. Find $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to\infty} f(x)$ to determine the end behavior of f.
5. Find and simplify $f'(x)$ and use $f'(x)$ to determine the intervals over which f is increasing or decreasing.
6. Find and simplify $f''(x)$ and use $f''(x)$ to determine the intervals over which f is concave up or down.
7. Graph $y = f(x)$. Label the intercepts, relative (local) extrema, inflection points, and vertical tangents.

HOMEWORK: Section 4.4: 9 - 57 every other odd.